

Calculating Standard Errors

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INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

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Statistical Significance

- Tells us if a particular difference is relatively large in relationship to an error (or uncertainty) expected by chance
- Depending on our interests, priorities and consequences of the decisions that would be made, the threshold to consider something large or small varies
- We usually assume a normal distribution of the error



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Statistical Significance

- It assumes that if we take two samples from populations that are equal, they will not always be exactly the same, but on average they will (Null Hypothesis)
- If the difference is “relatively” large, then we suspect these two samples were not drawn from populations that are equal (Alternative Hypothesis)



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Statistical Significance

- We say a difference is statistically significant when, based on a pre-established criteria, it is unlikely the observed difference is the result from samples taken from populations that are equal



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Statistical Significance

- Calculated dividing the difference of interest by the error, or uncertainty of the difference, and comparing it to a critical value or threshold
- If the difference is “small” relative to the error, we say it IS NOT statistically significant
- If the difference is “large” relative to the error, we say that it IS statistically significant



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Statistical Significance

- The criteria or threshold to say a difference is large or small depends on two main issues:
 - ✓ The directionality of our hypothesis
 - ✓ The tolerance level or acceptance



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Statistical Significance: Decisions

- Directionality
 - ✓ Decided based on the hypothesis you are investigating
 - ✓ Bilateral: Are there differences between A and B?
 - ✓ Unilateral: Is A larger than B?
- Tolerance or acceptance level
 - ✓ Given statistical theory about distribution of differences, how many times would I accept rejecting the null hypothesis and be wrong?
 - ✓ 1 out of 100? 5 out of 100?
 - ✓ What are the consequences of such mistake?



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Statistical Significance: Decisions

		Populations are Different	
		Yes	No
Did I find Differences?	Yes	Correct	Error
	No	Error	Correct



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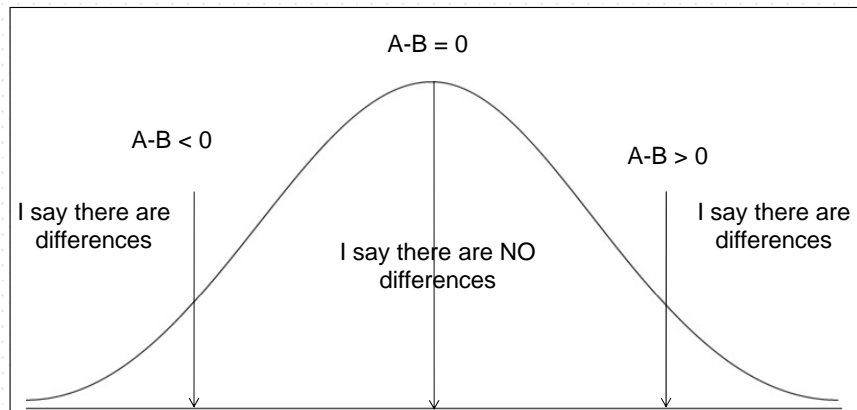
Statistical Significance: Decisions

		Directionality	
		One-tail	Two-tail
Probability of rejecting by error	More	+/- 1.64	+/- 1.96
	Less	+/- 2.33	+/- 2.58



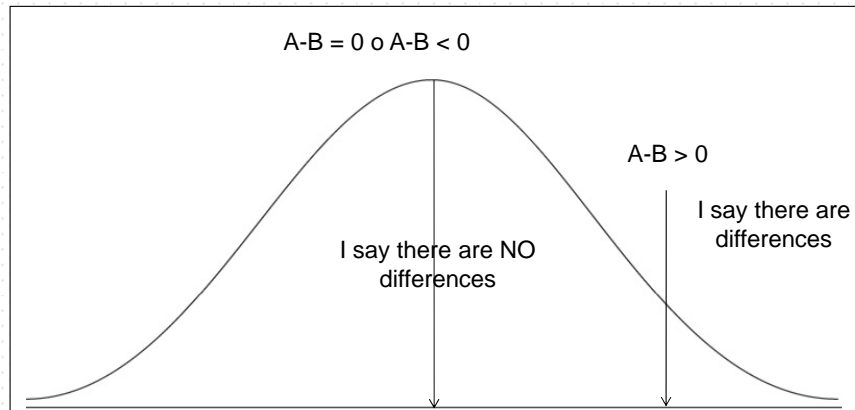
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Statistical Significance: Decisions



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Statistical Significance: Decisions



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Sources of Error

- Sample selection
 - ✓ Calculated using replications
 - ✓ Systematically change the overall contribution from different units within the sample, recalculate the results, and summarize according to formula
- Measurement
 - ✓ Calculated using plausible values
 - ✓ Compute the results with EACH plausible value, and calculate the variance of these estimates



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Sources of Error

- Some formulas...

$$SE_{\bar{\varepsilon}} = \sqrt{\left[\sum_{p=1}^P \left(f * \sum_{r=1}^R (\varepsilon_{r,p} - \varepsilon_{0,p})^2 \right) * \frac{1}{P} \right] + \left[\left(1 + \frac{1}{P} \right) * \frac{\sum_{p=1}^P (\varepsilon_{0,p} - \bar{\varepsilon}_{0,p})^2}{P-1} \right]}$$

Sampling Variance + Measurement Variance

- Where
 - ✓ When JK1, $f = (nreps - 1) / nreps$
 - ✓ When JK2, $f = 1.0$



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Types of Samples

- Independent
 - ✓ The units do NOT come from the same sampling frame
 - ✓ Errors are not correlated
 - ✓ Example:
 - Comparing results from one year to the next
 - Comparing results between countries



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Types of Samples

- Dependent
 - ✓ The units come from the same sampling frame
 - ✓ The errors are correlated
 - ✓ Example:
 - Comparing any variable within a country
 - Comparing results from a country with an international or composite estimate
 - ✓ We need to take the dependency into account



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Calculating Error of a Difference

- Between independent samples:

$$SE_{(a-b)} = \sqrt{SE_a^2 + SE_b^2}$$

- ✓ Calculate the statistics with corresponding error
- ✓ Use the formula above
- ✓ The comparison value is:

$$t = \frac{(a - b)}{SE_{(a-b)}}$$



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Calculating Error of a Difference

- Between dependent samples WITHOUT plausible values:

$$SE_{(a-b)} = \sqrt{\left[f * \sum_{r=1}^R ((a_r - b_r) - (a_0 - b_0))^2 \right]}$$

- Between dependent samples WITH plausible values:

$$SE_{(a-b)} = \sqrt{\left[\sum_{p=1}^P \left(f * \sum_{r=1}^R ((a_{r,p} - b_{r,p}) - (a_{0,p} - b_{0,p}))^2 \right) * \frac{1}{P} \right] + \left[\left(1 + \frac{1}{P} \right) \frac{\sum_{p=1}^P ((a_{0,p} - b_{0,p}) - (\bar{a}_{0,P} - \bar{b}_{0,P}))^2}{P-1} \right]}$$



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Calculating Error of a Difference

- In regression models we have that...

$$y = a + r_1 x$$

- When "x" takes on values of 1 or 0, we have the following property:
 - ✓ When $x=0$, we have that $y=a$
 - ✓ When $x=1$, we have that $y=a+(b-a)$
- For dependent samples, we can use the regression approach

$$SE_{(a-b)} = SE_{r_1}$$



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Calculating Error of a Difference

- Careful coding of categorical/grouping variables in combination with Regression statistics
 - ✓ Dummy Coding
 - ✓ Effect Coding
 - ✓ Contrast Coding



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Error of Difference with Composite

- The error of the difference between a national statistic and the international (composite) one is:

$$SE_{(\alpha-a)} = \sqrt{SE_{\alpha}^2 + \left(\frac{(C-1)^2 - 1}{C^2} \right) SE_a^2}$$

- We divide the difference by the error



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Thank you!

